Transmission Line Effect

Abstract

When an interconnection wire becomes sufficiently long or when the circuits become sufficiently fast, the inductance of the wire starts to dominate the delay behavior, and transmission line effect must be considered. This is more precisely the case when the rise and fall time of the signal become comparable to the time of the flight (t_{pline}) of the signal waveform across the line as determined by the speed of light.

Aim of this report is to show the transmission line behavior at different input resistance and output load (line terminations). Moreover, we compare the simulation made by the transmission line model with the waveform captured by the scope.

1 Transmission Line Model

The transmission line has the prime property that a signal propagates over the interconnection medium as a *wave*. In the wave mode, a signal propagates by alternatively transferring energy from the electric to magnetic fields, or equivalent from the capacitance to the inductive modes. A distributed RLC model of a wire, known as a transmission line model, becomes the most accurate approximation of the actual behavior.

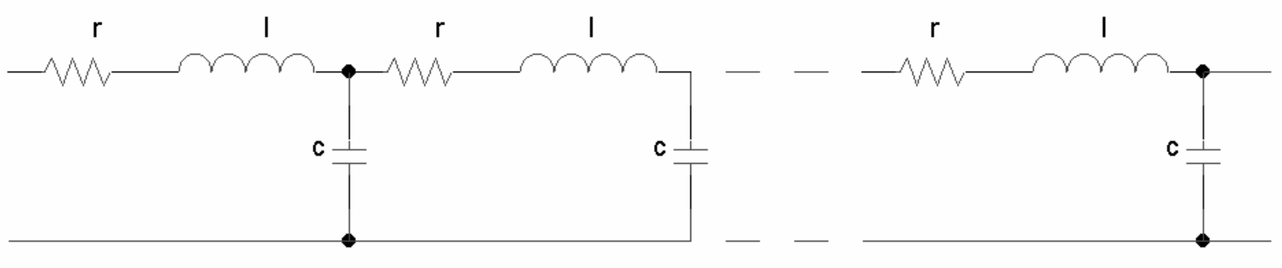

Figure 1: Transmission line model

r, *c* and *l* are the resistance, capacitance and inductance per unit length, respectively.

Due to the high conductivity of the interconnection material (copper), the resistance of the transmission line can be ignored, and a simplified capacitive/inductive model, called the *lossless transmission line*, is appropriate.

A step input applied to a lossless transmission line propagates along the line with a speed ν,

$$\nu = \frac{1}{\sqrt{l \cdot c}}$$

The propagation delay per unit length (t_{pline}) of a transmission line is the inverse of the speed:

$$t_{pline} = \sqrt{l \cdot c}$$

The impedance, called the *characteristic impedance* (Z_0) of the line, is a function of the dielectric medium and the geometry of the conducting wire and isolator:

$$Z_0 = \sqrt{\frac{l}{c}}$$

Typical values of Z_0 for IC interconnect ranges from 50 to 100 Ω.

It's possible define a rule of thumb to determinate when the transmission line effect should be considered:

Equation 1

$$t_r(t_f) < 2.5 t_{pline} = 2.5\frac{L}{v}$$

L is the total length of the interconnection wire.
Transmission line effect should be considered when rise or fall time of the input signal (t_r, t_f) is smaller than the time-of-flight of the transmission line (t_{pline}). This one is the time takes for the wave to propagate from one end of the wire to the other.

2 Termination

The behavior of the transmission line is strongly influenced by the termination of the line. The termination determines how much of the wave is reflected upon arrival at the wire end. This is expressed by the *reflection coefficient* ρ that determines the relationship between the voltage and currents of the incident and reflected waveform.

$$\rho = \frac{V_{refl}}{V_{inc}} = \frac{I_{refl}}{I_{inc}} = \frac{R - Z_0}{R + Z_0}$$

Where R is the value of the termination resistance.
The total voltages and currents at the termination end are the sum of incident and reflected waveform.

$$T = 1 + \rho$$
$$V = V_{inc}(1 + \rho)$$
$$I = I_{inc}(1 - \rho)$$

T is the *transmission coefficient*.
The transient behavior of a complete transmission line is influenced by the characteristic impedance of the line, the series impedance of the source Z_S and the loading impedance Z_L at the destination end.

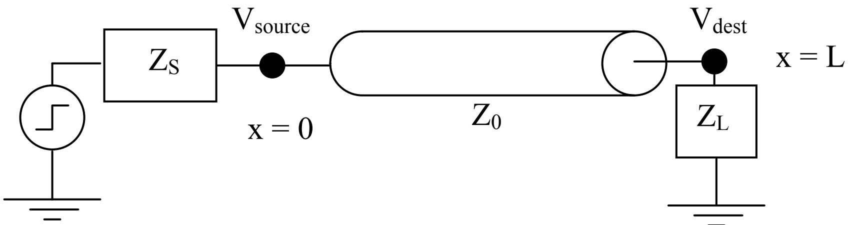

Figure 2: Transmission line with terminating impedance.

Figure 2 shows a real case when a transmission line is used to propagate a signal. In the first case (Z_L = Z_0) the terminating resistance is equal to the characteristic impedance of the line. The termination appears as an infinite extension of the line and no waveform is reflected. This is also demonstrated by the value of ρ, which equal 0.
In the last case, the line termination is an open circuit ($R = \infty$) and $\rho = 1$. The total voltage waveform after reflection is twice the incident one.
Figure 3 compare these case: the scope trace looks the same, but the vertical scale of y-axis in the left figure is double that right one.

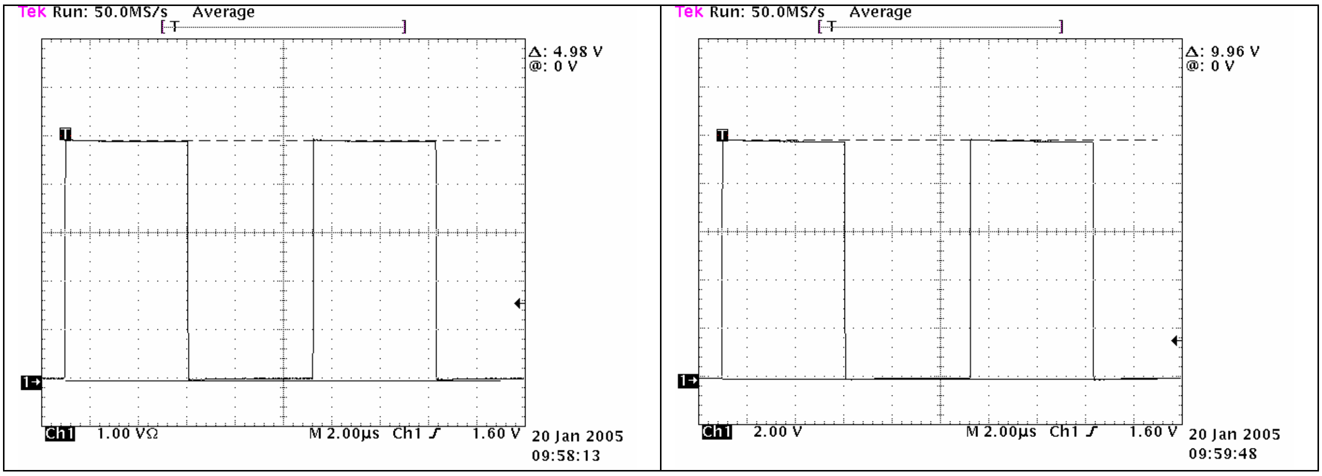

Figure 3: right – scope input impedance adapted to 50 Ω; left – scope input impedance equal to 1 MΩ.

Consider the case where the wire is open at the destination end, or $Z_L = \infty$, and $\rho_L = 1$. An incoming wave is completely reflected without phase reversal.
An important technique to calculate the reflection along a transmission line is the lattice diagram: the next section explains this tool.

3 Lattice Diagram

The lattice diagram is a technique to simplify the accounting of reflections and waveforms. The diagram contains the values of the voltage at the source and destination ends, as well as the values of the incident and reflected waveforms. The line voltage at a termination point equals the sum of the previous voltage, the incident and reflected waves.

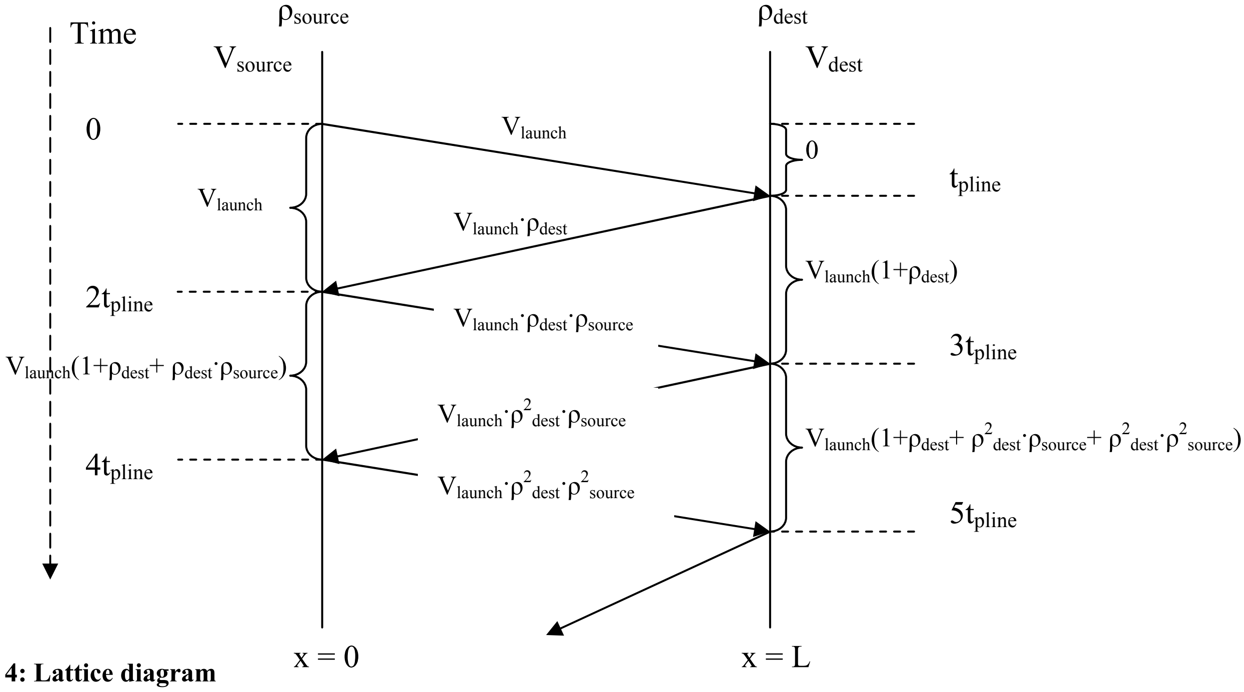

Figure 4: Lattice diagram

The lattice diagram shows the transmission line boundaries (x = 0, x = L), the reflection coefficient (ρ_{source} and ρ_{dest}) and the line voltage after a propagation delay time. The time axis is shown vertically, while the arrows indicate the signal propagation across the line.
There are three steps for solving all transmission line problem:

1. Determination of launch voltage and final dc voltage: the behavior of the transmission line makes the launch and final voltage easy to calculate because it's simply a voltage divider $V_{launch} = \frac{Z_0}{Z_0 + Z_S} V_{in}$, $V_{final} = \frac{Z_L}{Z_L + Z_S} V_{in}$.
2. Calculation of the destination reflection coefficient and the voltage delivered to the load. The transient behavior of transmission line delays the arrival of launched voltage until time $t = t_{pline}$. The equations to calculate reflection coefficient at destination, reflected and destination voltage are respectively: $\rho_{dest} = \frac{Z_L - Z_0}{Z_L + Z_0}$, $V_{reflected} = \rho_{dest} \cdot V_{incident}$ and $V_{dest} = V_{reflected} + V_{incident}$
3. Calculation of the source reflection coefficient and resultant source voltage. The transient behavior of transmission line delays the arrival of voltage reflected from the load until time $t = 2t_{pline}$. The equations to calculate reflection coefficient at source, reflected and source voltage are respectively: $\rho_{source} = \frac{Z_S - Z_0}{Z_S + Z_0}$, $V_{reflected} = \rho_{source} \cdot V_{incident}$ and $V_{source} = V_{launch} + V_{reflected} + V_{incident}$

It's possible to repeat these steps in order to calculate other destination/source reflection voltage.

4 Simulations and Measurements

In order to show the behavior of the transmission line, it's possible to feed a long coaxial cable (L = 206 cm) with a square wave (f = 100 kHz; Duty cycle D = 50%; $t_r = t_f = 0.9$ ns; $V_{display} = 5$ V → $V_{in} = 10$ V) and to measure V_{soure} and V_{dest}. In this condition it's possible to calculate the propagation delay time along the line by the equations express in the section 1 ($t_{pline} = 10.3$ ns [1]) and to verify the

Equation 1: $(t_r)\, 0.9 < 2.5 \cdot 10\, (t_{pline})$. All test are executed with $Z_L = \infty$ and $\rho_L = 1$

4.1 Large source resistance

In this test, the source resistance is bigger ($Z_S = 320\ \Omega$) than characteristic impedance of the line ($Z_0 = 50\ \Omega$).
Only a small fraction of the incoming signal V_{in} is injected into the transmission line. The amount injected is determined by the resistive divider formed by the source resistance and the characteristic impedance Z_0.

$$V_{source} = V_{launch} = \frac{Z_0}{Z_0 + R_S} V_{in} = 1.35 \text{ V}$$

[1] For a 50 Ω coaxial cable, the inductance per unit length l equal 2.5 nH/cm, while the capacitance per unit length c equal 1 pF/cm

This signal reaches the end of the line after L/v sec and is fully reflected, which effectively doubles the amplitude of the wave (V_{dest} = 2.7 V). Approximately the same happens when the wave reaches the source node again. The incident waveform is reflected with amplitude determined by the source reflection coefficient, which equals 0.73 for this particular case:

$$\rho_{source} = \frac{Z_S - Z_0}{Z_S + Z_0} = 0.73$$

The voltage amplitude at source and destination nodes gradually reaches its final value of V_{in}. The overall rise time is, however, many times L/v. The lattice diagram is shown on Figure 5.

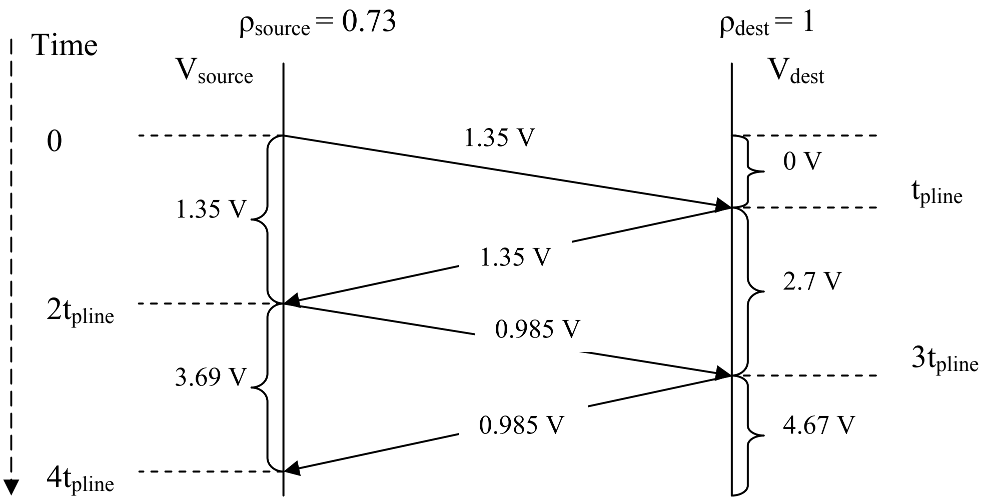

Figure 5: Lattice diagram for the large source resistance test.

Figure 6 reports the transmission line behavior for this particular case: right picture shows the entire transient, while the left one shows a zoom on x-axis.

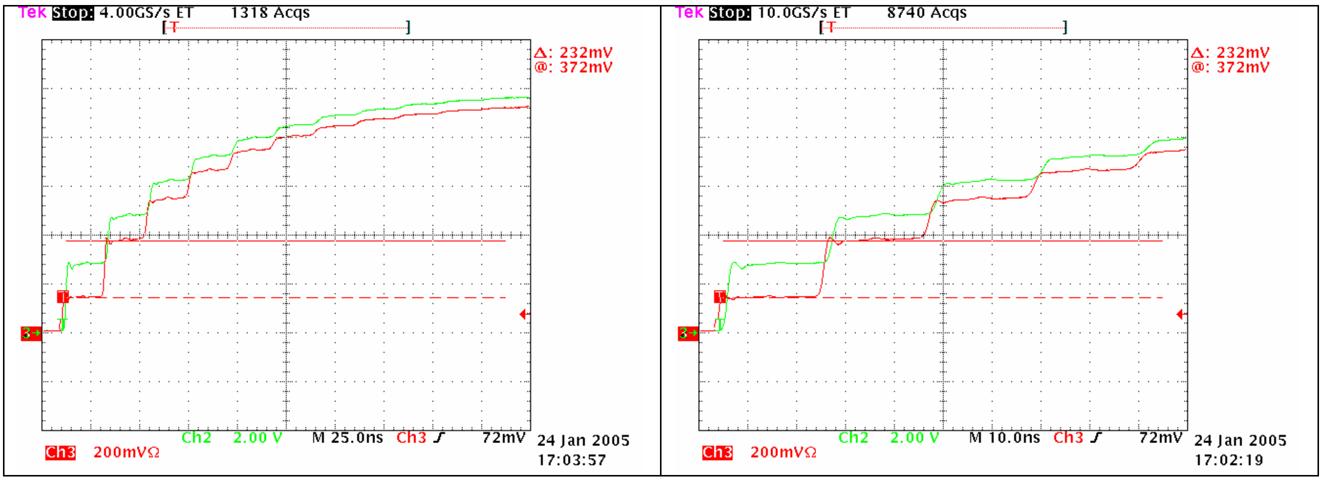

Figure 6: Large source resistance: picture create by the scope. Left picture is a zoom of the right one.

The red line (bottom trace) is captured by an active probe and reports the source voltage, while the green line (top trace) is the destination voltage (See Figure 2 for more details). Unfortunately, the active probe uses to capture V_{source} adds a time delay, so that the red line is shifted to right about 10 ns (i.e. the active probe delay).

It's possible to see the correct delay time between source and destination signal by a software simulator and Figure 7 shows the traces recorded.
Comparing Figure 5, Figure 6 and Figure 7, it's possible to agree that lattice diagram, simulation data and scope trace are perfectly aligned.

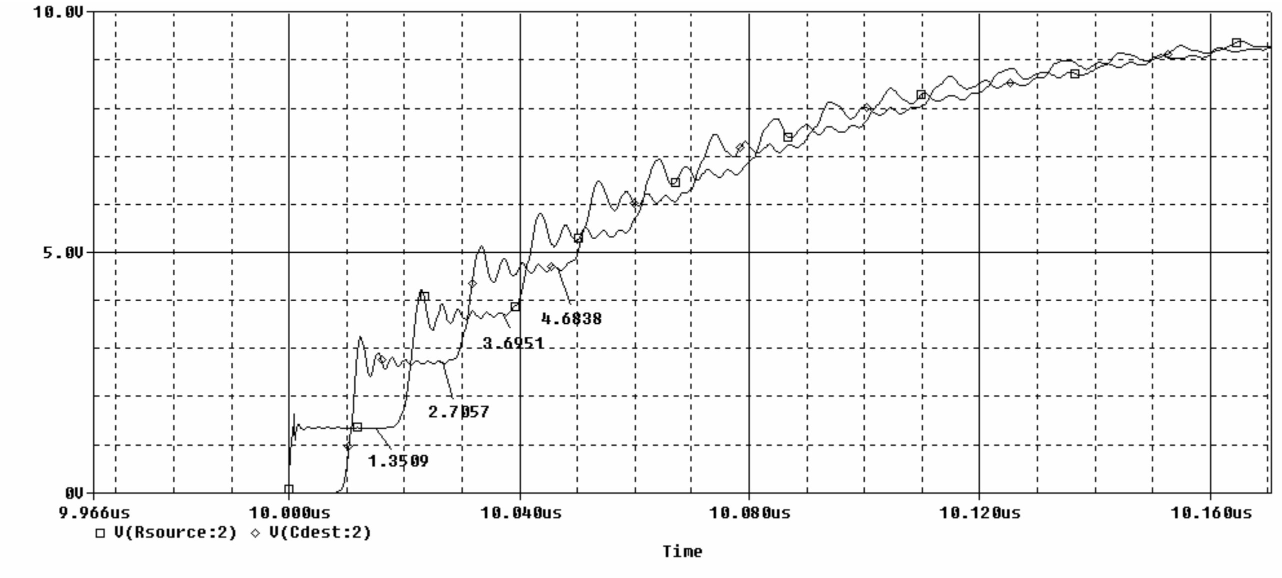

Figure 7: Source and destination voltage make by PSpice simulator.

The schematic diagram simulated by PSpice is shown on Figure 8. The transmission line is drown as the model described in the section 1: it's shared in 20 basic subcircuits (each subcircuits models a small transmission line 10 cm long) with capacitance and inductance multiplied per 10 in order to have always a characteristic impedance equal to 50 Ω.

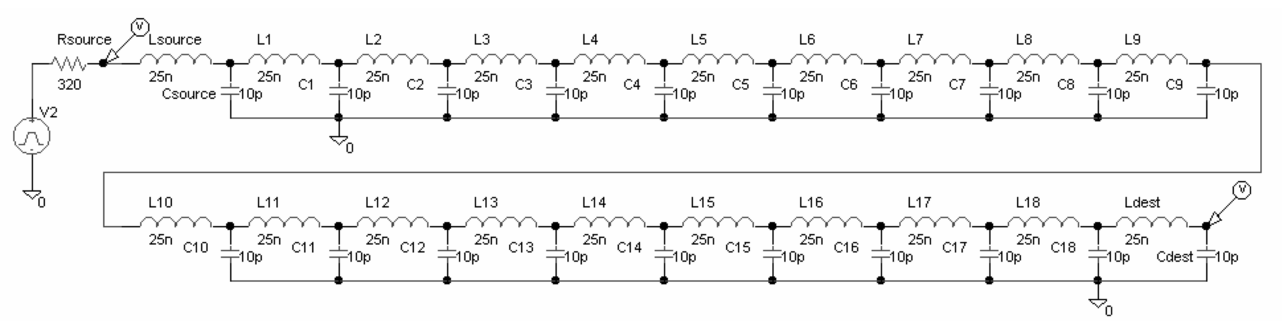

Figure 8: Transmission line simulates by PSpice.

Figure 9 shows the simulation results when the transmission line is modelled with only five basic subcircuits: the traces recorded are very different respect to Figure 6 (traces realized by the scope) and Figure 7 (traces recorded by PSpice simulator with 20 basic subcircuits): a transmission line is modelled correctly when it's drown with many small basic subcircuits.

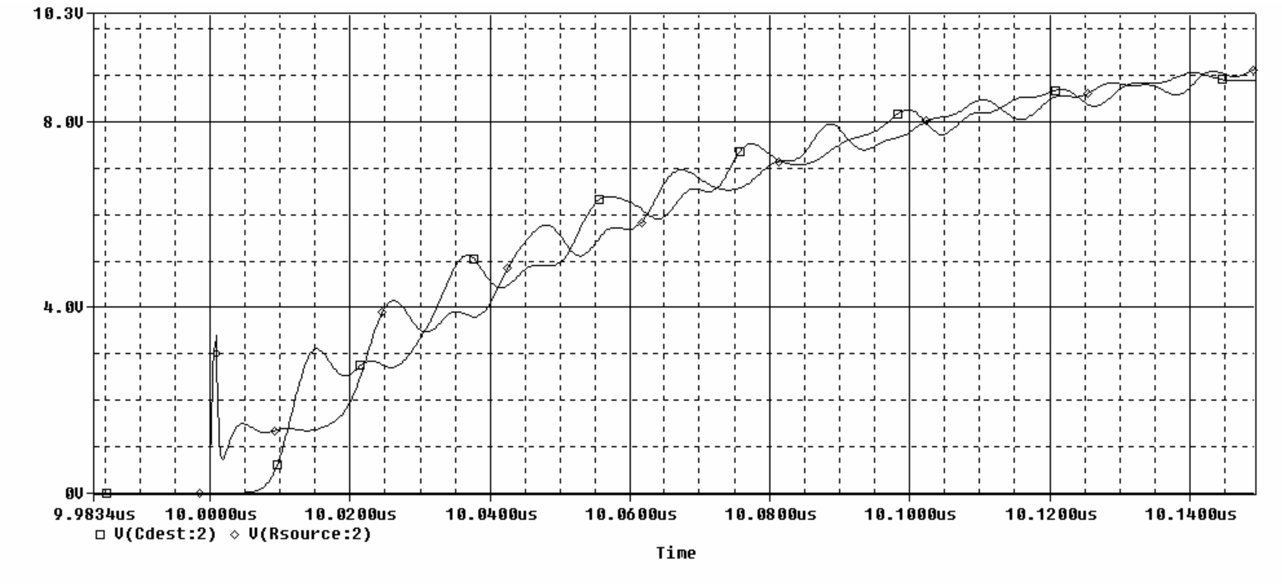

Figure 9: Simulation runs with five transmission line basic subcircuits.

4.2 Small source resistance

In this test, the source resistance is smaller (Z_S = 8.33 Ω) than characteristic impedance of the line (Z_0 = 50 Ω).
A large portion of the input is injected in the line. Its value is double at the destination end, which causes a severe overshoot. At the source end, the phase of the signal is reversed (ρ_S = -0.714). The signal bounces back and forth and exhibits severe ringing. It takes multiple L/ν before it settles.
In order to realize an input series resistance smaller than characteristic impedance, it's possible to connect 10 Ω resistor at the function generator output.

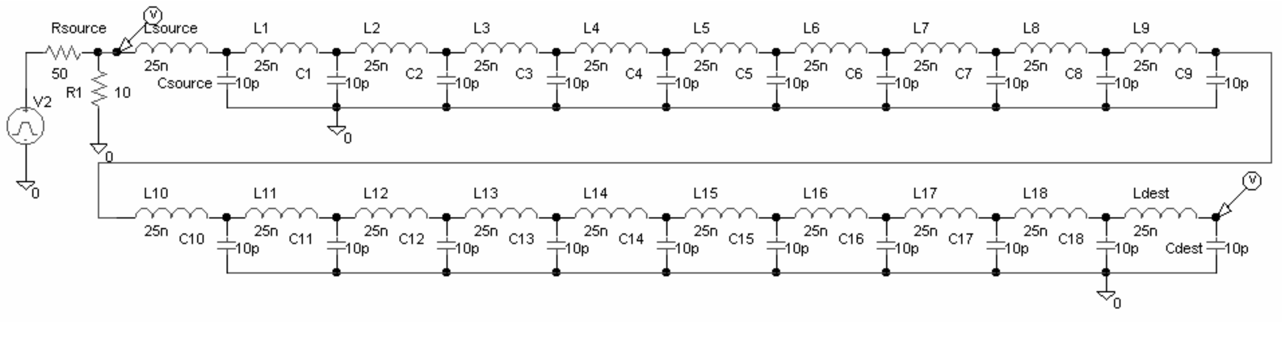

Figure 10: Transmission line with small source resistance simulates by PSpice.

The transmission line model is equal to large source resistance; it's only added a 10 Ω resistor after V2 output resistor.
The Thevenin equivalent circuit for the input stage is shown on Figure 11.

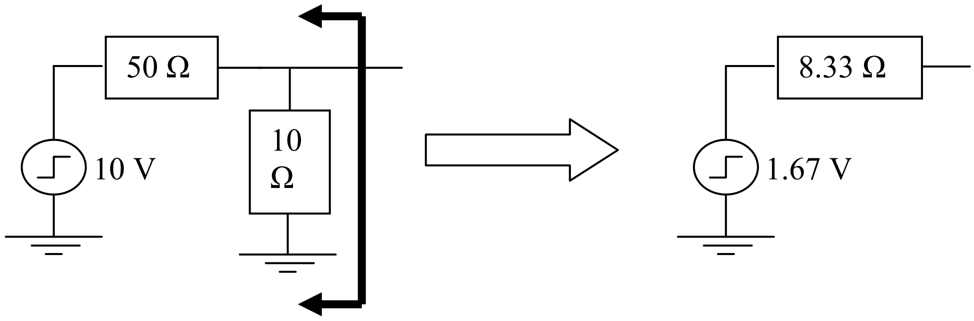

Figure 11: Thevenin equivalent circuit for the small input resistance.

The lattice diagram can be calculated: V_{launch} is equal to $V_{launch} = 1.67\frac{50}{50+8.33} = 1.43\text{ V}$ and V_{final} = 1.67 V.

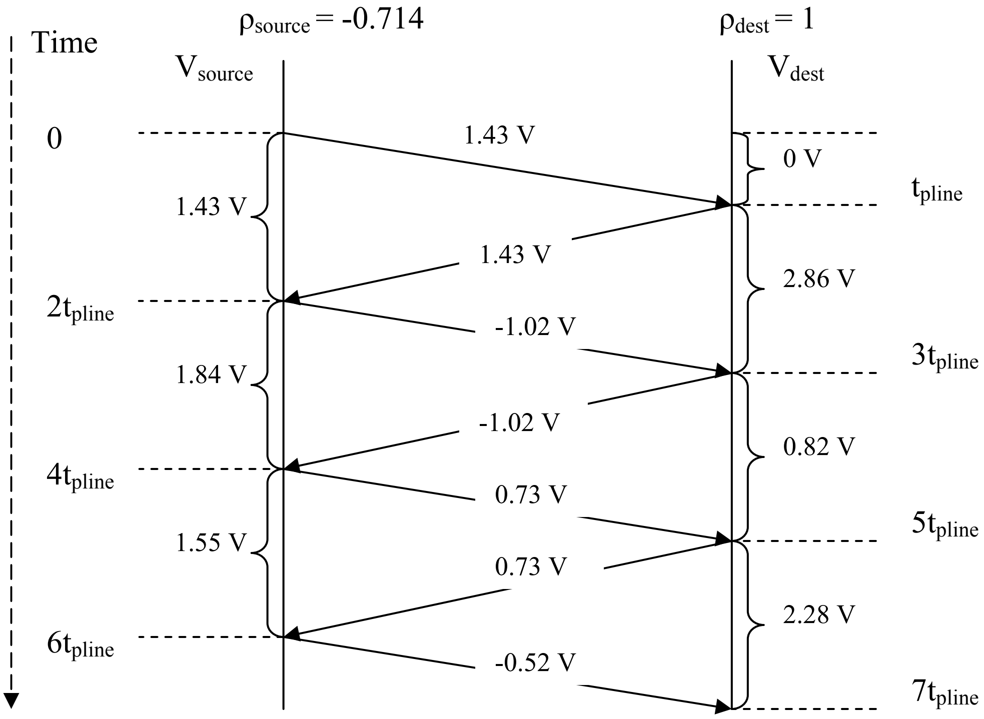

Figure 12: Lattice diagram for the small source resistance test.

In this case, the ρ_{source} is a negative number, so that there is a phase inversion when the wave arrives at the source end. This condition builds voltage overshoots at the source and destination ends, as it's possible to see on Figure 13.

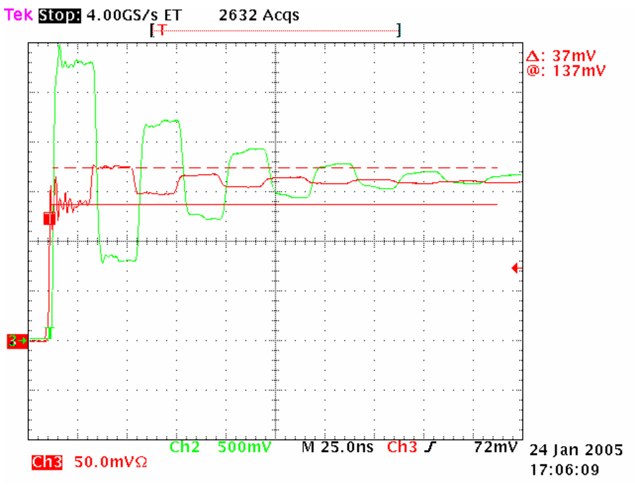

Figure 13: Small source resistance: picture creates by the scope.

The simulations recorded for the small source resistance test are reported on Figure 14.

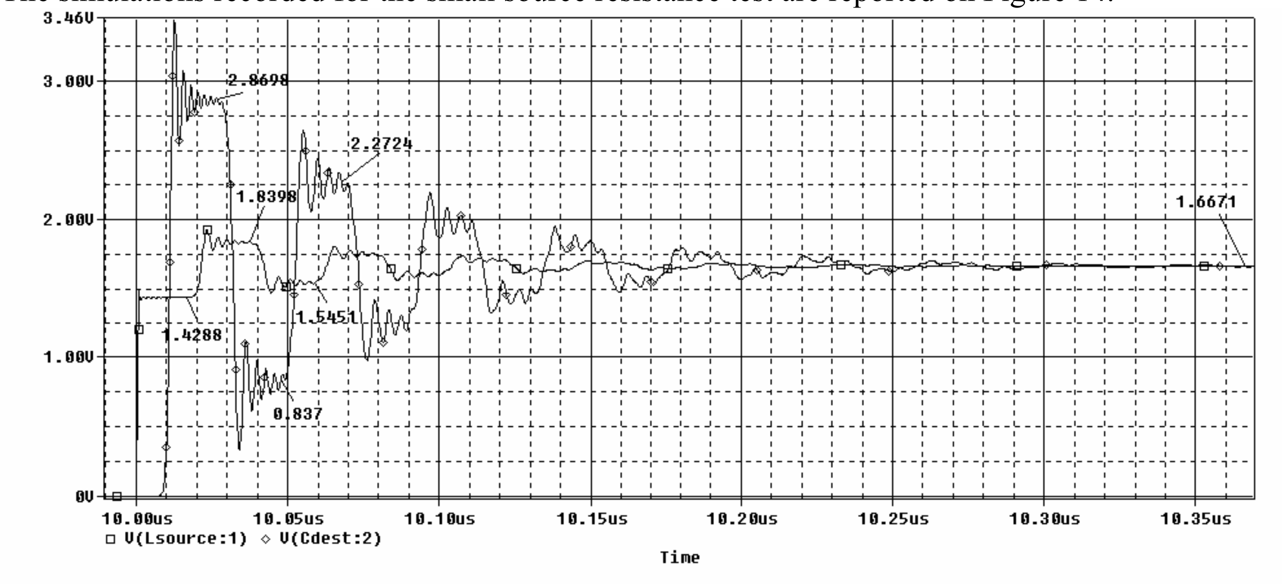

Figure 14: Source and destination voltage for small source resistance test make by PSpice simulator.

4.3 Matched source resistance

When the source resistance and the characteristic impedance of the line are equal, half of the input signal is injected at the source. The reflection at the destination end doubles the signal, so that the final value is reached immediately. It is obvious that this is the most effective case. Matching the line impedance at the source end is called *series termination* ($\rho_{source} = 0$). Note that the above analysis is an ideal one, as it assumed that the signal has a zero rise time. In real condition the signal are substantially smoother.

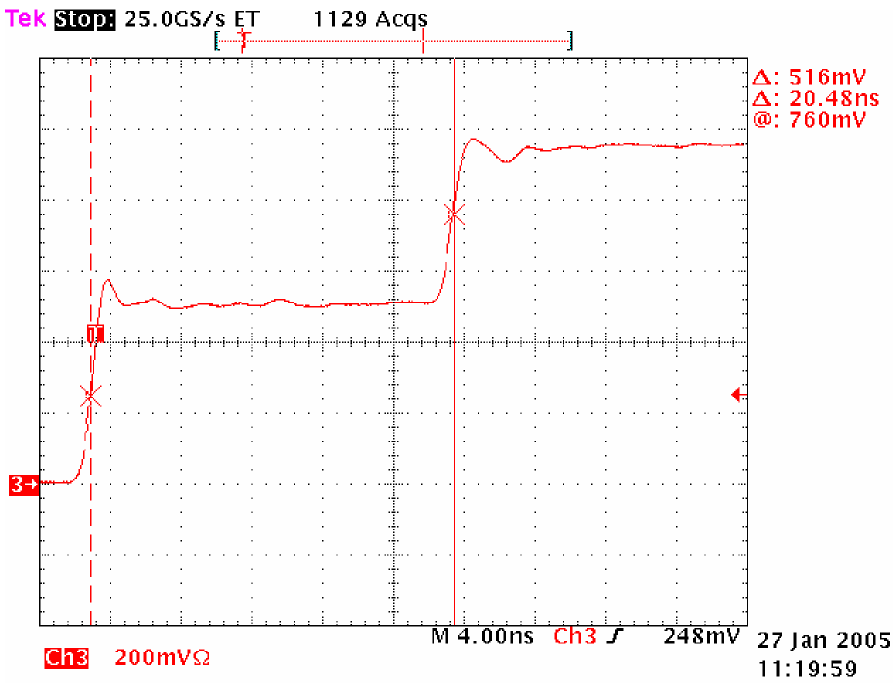

Figure 15: Adapted source resistance: picture creates by the scope.

Figure 15 reports the picture recorded by the scope. By the scope cursors, it' possible to calculate 2_{tpline} (20.48 ns), while Figure 16 shows same measure make by software simulator (20.76 ns). By the theory explains in section 1, it's easy to calculate this delay time: $2t_{pline} = 2 \cdot L\sqrt{l \cdot c} = 20.6\,\text{ns}$.

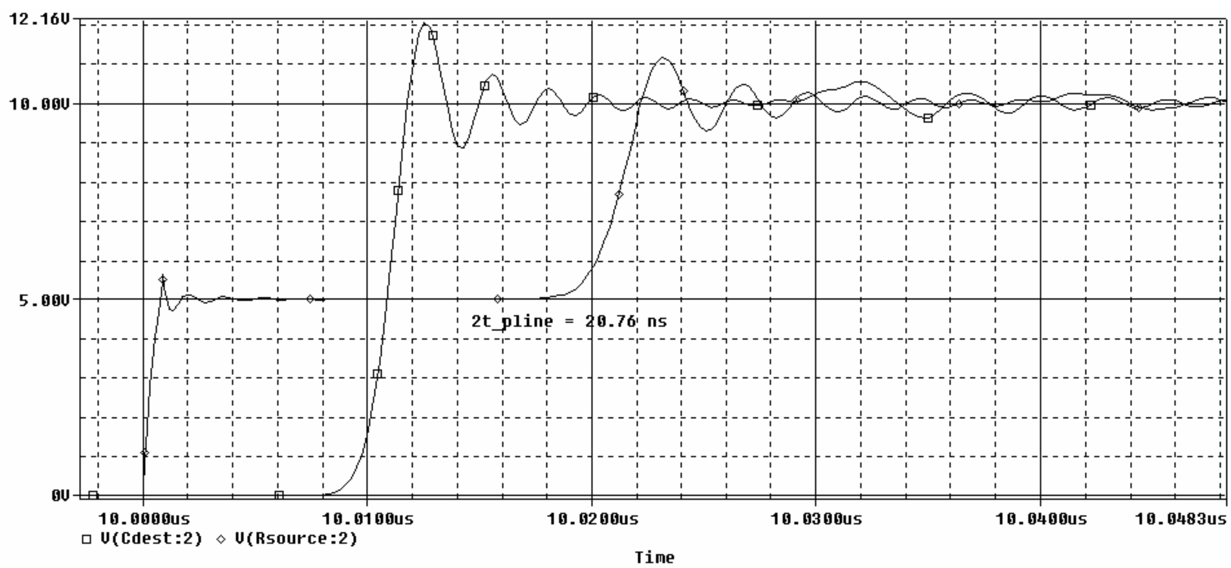

Figure 16: Source and destination voltage for adapted source resistance test make by PSpice simulator.

4.4 Capacitive termination

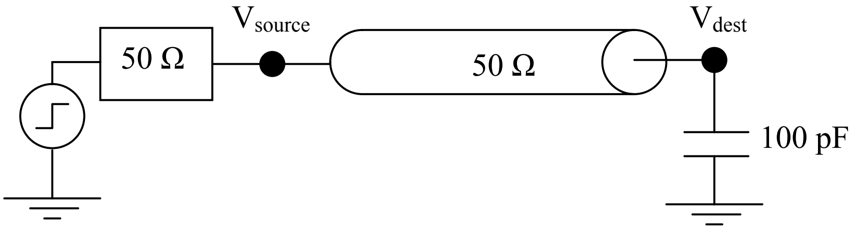

Figure 17: Transmission line terminates with a capacitive load.

The characteristic impedance of the transmission line determines the current that can be supplied to charge capacitive load C_L (C_L = 100 pF in the following test). From the load's point of view, the line behaves as a resistance with value Z_0. The transient response at the capacitor node, therefore, displays a time constant Z_0C_L.

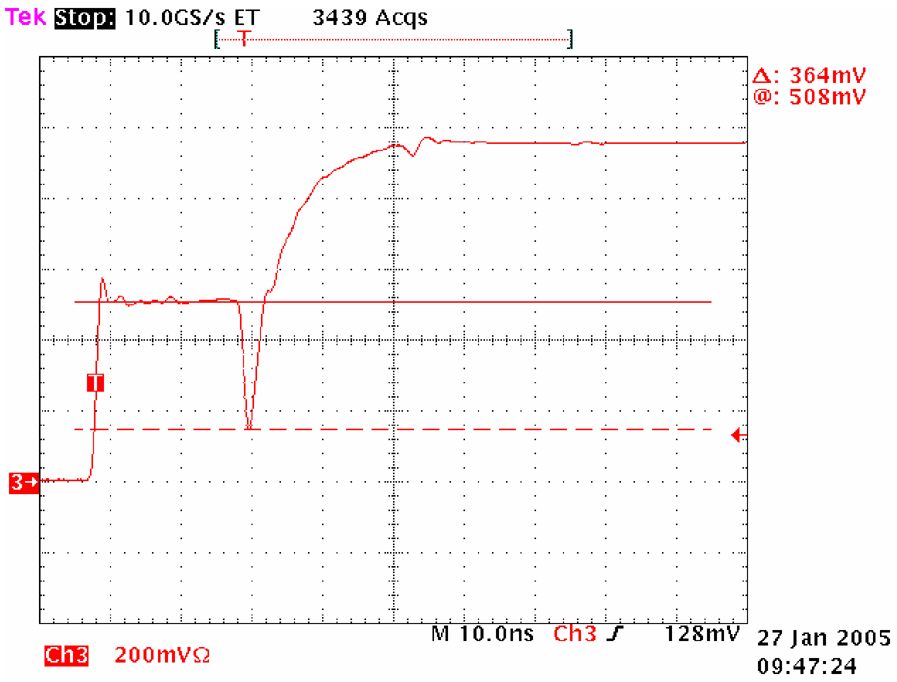

Figure 18: Capacitively terminated transmission line.

After $2t_{pline}$ an unexpected voltage drop occurs at the source node that can be explain as follows. Upon reaching the destination node, the incident wave is reflected. This reflected wave also approaches its final value asymptotically. Since V_{dest} equal 0 initially instead of the expected jump to 10 V, the reflection equals –5 V rather than the expected 5V. This force the transmission line temporarily to 0 V, as shown in Figure 18. This effect gradually disappears as the output node converges to its final value.
Figure 18 shows the waveform recorded by the simulator.

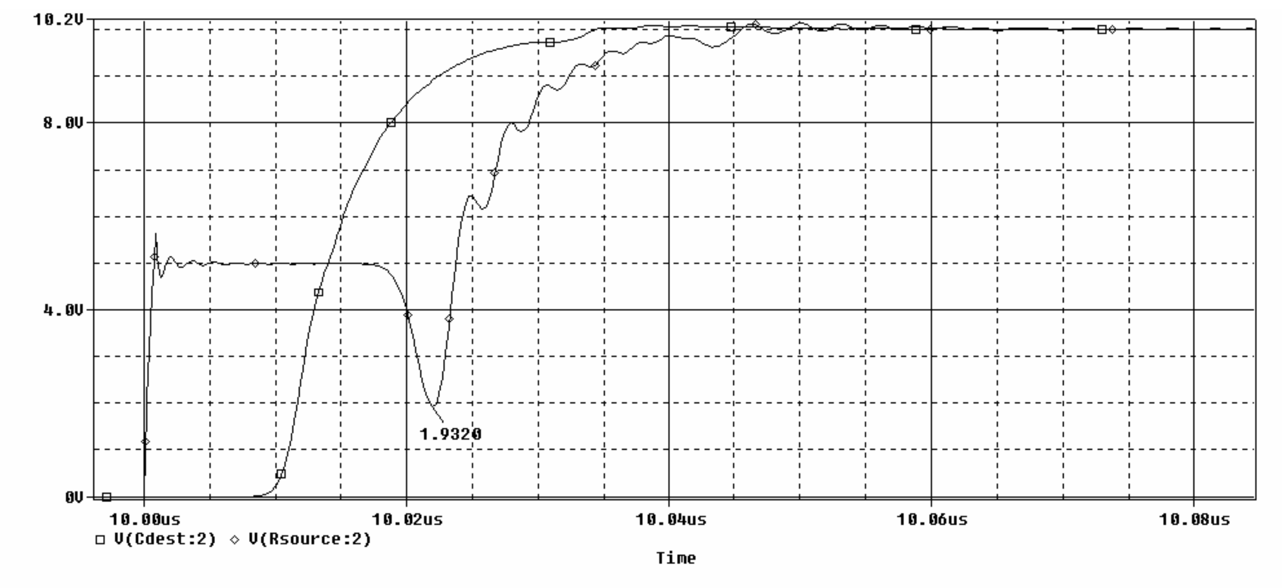

Figure 19: Capacitively terminated transmission line simulates by Psipce.

4.5 Series of two transmission line with different Z_0

The circuit describes in this section is shown on Figure 20

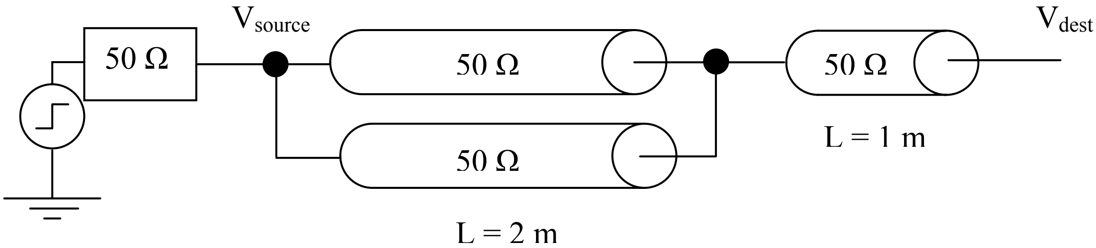

Figure 20: Series of two transmission line: circuit under test.

When two transmission lines are joined together in parallel mode, the equivalent characteristic impedance is $Z_{01} = Z_0/2$. This result can be explained observing the relation among Z_0, l and c: $Z_0 = \sqrt{l/c}$. The new impedance per unit length 'l_n' is equal to $l_n = l / 2$, while the new capacitance per unit length 'c_n' is double $c_n = 2$ c, such as a parallel of inductance and capacitance respectively. The new characteristic impedance is equal to:

$$Z_{01} = \sqrt{\frac{l_n}{c_n}} = \sqrt{\frac{l/2}{2c}} = \frac{1}{2}\sqrt{\frac{l}{c}}\,.$$

If the parallel transmission lines are *n*, then the new characteristic impedance is equal to $Z_{0n} = \frac{1}{n} Z_0$.

The propagation delay time of the parallel transmission line is:

$$t_{pline1} = \sqrt{l_n \cdot c_n} = \sqrt{l/2 \cdot 2c} = \sqrt{l \cdot c}$$

It's easy to verify that the propagation delay along two parallel transmission lines doesn't change.

The circuit under test has two transmissions line with different characteristic impedance Z_0: the first one gets $Z_0 = 25\ \Omega$ and 2 meter long, while the second one gets $Z_0 = 50\ \Omega$ and 1 meter long.

Following there are listed the source/destination reflection coefficients and initial/final voltage for this test:

- $V_{lauch} = 10\frac{25}{50+25} = \frac{10}{3}$ V
- $\rho_{source} = \frac{50-25}{50+25} = \frac{1}{3}$
- $\rho_{dest1} = \frac{50-25}{50+25} = \frac{1}{3}$
- $\rho_{source1} = \frac{25-50}{50+25} = -\frac{1}{3}$
- $\rho_{dest} = 1$
- $V_{final} = 10$ V

Lattice diagram can calculated the reflection at source and destination ends, as shown on Figure 21.

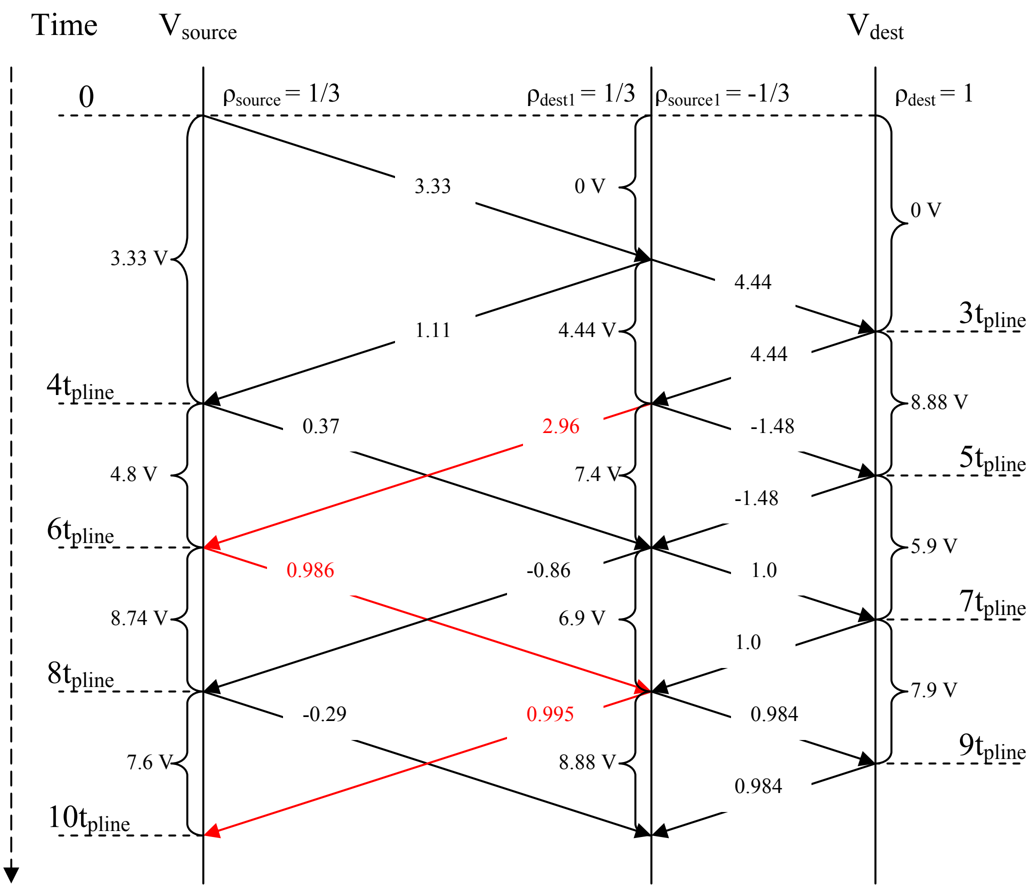

Figure 21: Series of two transmission line with different characteristic impedance: lattice diagram.

The t_{pline} shows in the lattice diagram is the delay time necessary to waveform to propagate along 1 meter transmission line (the second transmission line shows on Figure 20).

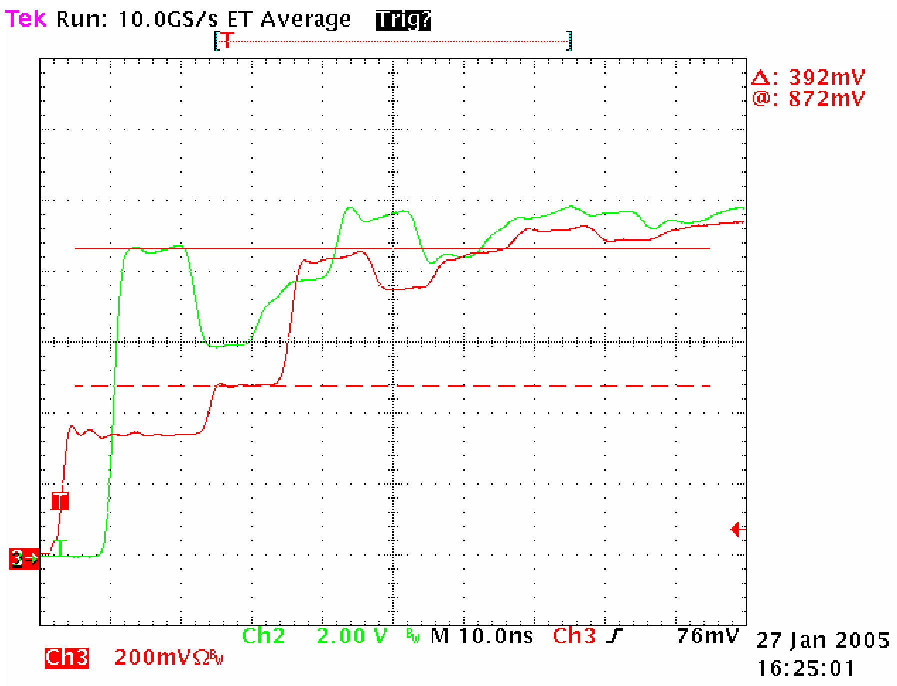

Figure 22: Series of two transmission line: picture from scope.

It's possible to verify the values calculate from lattice diagram with the picture recorded by the scope shows on Figure 22.

5 Conclusion

*To avoid potentially disastrous transmission line effects such as ringing or slow propagation delays, the transmission line should be terminated, either at the source (**series terminations**), or at the destination (**parallel termination**) with a resistance equal to the characteristic impedance Z_0 of the line.*

6 References

1. J. M. Rabay; *"Digital Integrated Circuits"*, Prentice Hall International